國立政治大學
112 學年度

統計學系碩士班甄試考試試題

考試科目	統計學	所(組)別	統計學系	考試時間	111年11月12日 星期六 10：00－11：40

注意：本科目有四份試題，第一份試題答案寫在第一卷答案本上，依此類推。每份試題配分為 25 分，合計共 100 分。

第一份試題(共 25 分，答案請寫在第一卷答案本上)

1. (10 points) Suppose that the number of calls per hour received at a hotel's help desk follows a Poisson distribution with mean $\lambda = 2$. Suppose that 80% of the calls are for room reservations. Find the probability that exactly one call is received for room reservations within two hours.

2. (15 points) Bagel Kitchen produces bagels by two machines: I and II. It is known that machine I produces 40 percent of the bagels, while machine II produces 60 percent of the bagels. Suppose that the weights (grams) of bagels by machine I and machine II follow $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively, where $(\mu_1, \sigma_1) = (105, 5)$ and $(\mu_2, \sigma_2) = (110, 2)$. Let X denote the weights of bagels produced in Bagel Kitchen. Denote by $\phi(z; \mu, \sigma)$ the probability density function (pdf) of a normal distribution with mean μ and standard deviation σ. Find the pdf of X; find also the conditional probability that a bagel is produced by machine I, given that it weighs 105 grams.

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第二份試題(共 25 分，答案請寫在第二卷答案本上)

1. Suppose that 100 observations are drawn from a normal population whose standard deviation is 200, produced a mean of 980. At 99% confidence, you want to determine whether the mean of the population from which this sample was taken is significantly different from 1,000.
   (a) State the null and alternative hypotheses. (2 分)
   (b) Specify the rejection region. (3 分)
   (c) Compute the *p*-value. (3 分)
   (d) Interpret the results. (2 分)
   (e) Compute Type II error β . (assume that mean = 948.5). (5 分)

   附表於 5,6 頁

Multiple Choice:

2. For a given level of significance, if the sample size increases, the probability of a Type II error will: (2 分)
   (a) remain the same (b) increase (c) decrease (d) be equal to 1.0
3. Which of the following statement is not true? (2 分)
   (a) The probability of making a Type II error increases as the probability of making a Type I error decreases
   (b) The probability of making Type II error and the level of significance are the same
   (c) The power of the test decreases as the level of significance decreases
   (d) All of the above statements are not true
4. In hypothesis testing, whatever we are investigating or researching is specified as: (2 分)
   (a) the null hypothesis
   (b) the alternative hypothesis
   (c) either the null or alternative
   (d) *p*-value
5. In order to determine the *p*-value, which of the following is not needed? (2 分)
   (a) The level of significance
   (b) Whether the test is one or two tail
   (c) The value of the test statistic
   (d) All of the above are needed
6. The power of a test is the probability of making: (2 分)
   (a) A correct decision when the null hypothesis is false
   (b) A correct decision when the null hypothesis is true
   (c) Incorrect decision when the null hypothesis is false
   (d) Incorrect decision when the null hypothesis is true

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第三份試題(共 25 分，答案請寫在第三卷答案本上)

計算題證明題。請寫出計算過程，否則不予計分。

1. (7%) Suppose that the random variables $Y_1, Y_2, \ldots, Y_n$ satisfy

$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \qquad i = 1, 2, \ldots, n,$

where $x_1, \ldots, x_n$ are fixed constants; $\varepsilon_1, \ldots, \varepsilon_n$ are i.i.d. $N(0, \sigma^2)$; $\sigma^2 > 0$ is an unknown parameter. Find the maximum likelihood estimator of $(\beta_0, \beta_1, \sigma^2)$.

2. A university admission committee aimed to investigate whether high school GPA (x_1), the verbal score on the Scholastic Aptitude Test (SAT) (x_2), and the mathematics score on the SAT (x_3) would be good predictors for the GPA in the university (Y). They conducted a multiple regression analysis

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

based on 9 observations, and the null hypothesis of the F test was rejected.

Predictor	coefficient	SE coefficient	t
Constant	−.411	.782	
x_1	1.201	.296	(*)
x_2	.002	.002	
x_3	−.002	.002	

(a) (2%) Find the (*) in the above table.

(b) (6%) Consider the t test to determine the significance of β_1. At a .05 level of significance, what is your conclusion for the test? Interpret the $\hat{\beta}_1$.

Note: Some upper critical values of the t distribution are in the followings:

$t_{\mathrm{df}=7,\alpha=.025} = 2.365 \quad t_{\mathrm{df}=7,\alpha=.05} = 1.895 \quad t_{\mathrm{df}=9,\alpha=.025} = 2.262 \quad t_{\mathrm{df}=9,\alpha=.05} = 1.833$

3. Taylor wanted to compare the flight times between Los Angeles and San Diego on three airlines, AA, UA, and Delta. She randomly selected six flights and recorded the flight times (in minutes) for each of the three airlines. Define x_{ik} be the observation of the i−th replicate for the k−th treatment, where $k = 1, 2,$ and 3 stand for AA, UA, and Delta, respectively. Further define $\bar{x}_{\cdot k} = \sum_{i=1}^{6} x_{ik}/6$. Based on data, we have $\bar{x}_{\cdot 1}$ =52, $\bar{x}_{\cdot 2}$ =54, $\bar{x}_{\cdot 3}$ =58, $\sum_{i=1}^{6} {x_{i1}}^2$ =16418, $\sum_{i=1}^{6} {x_{i2}}^2$ =17570, and $\sum_{i=1}^{6} {x_{i3}}^2$ =20340.

(a) (8%) Suppose that each of the three populations follows a Normal distribution. Set up the ANOVA table to determine whether there is a difference in the mean flight times among the three airlines.

(b) (2%) Using a 0.05 significance level, what is the conclusion about the mean comparison in (a)?

Note: Define the critical value(s) you need using the notation $F_{\mathrm{df1,df2},\alpha}$, and specify df1, df2, and α.

Express your conclusion in terms of $F_{\mathrm{df1,df2},\alpha}$.

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第四份試題(共 25 分，答案請寫在第四卷答案本上)

1. (10%) The following table classifies students by field of study and whether or not they have a loan. The students are randomly selected from a large university.

Field of study	**Student loan**	
Agriculture	32	35
Engineering	98	137
Management	24	51
Science	31	29
Liberal arts	89	124

Conduct a test to examine whether the proportions of students having a loan in different fields are equal at the significance level of 0.05.

2. (15%) The same randomly selected students in the previous question are asked to respond some questions regarding their interests and attitudes, which forms a score to measure altruism, an interest in the welfare of others. Each Student is classified as low, medium, or high on this score.

	Score		
Field of study	**Low**	**Medium**	**High**
Agriculture	8	27	32
Engineering	12	129	94
Management	10	40	25
Science	7	29	24
Liberal arts	7	77	129

Test whether there is an association between the score and field of study at the significance level of 0.05.

附表於 5,6 頁

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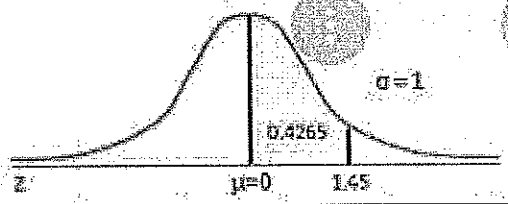
附表 1

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

備考：試題隨卷繳交

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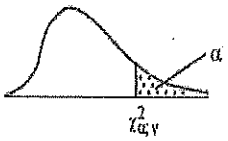
附表 2

Table of the Chi-square Distribution



α =	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	=α
V = 1	0.0000393	0.000157	0.000628	0.000982	0.00393	0.0158	0.0642	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	V = 1
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	90.405	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

備考：試題隨卷繳交