博士班 線性模式 學科考試 2/18/95 第1

1. (a) Let λ be the likelihood ratio test statistic for testing that $\underset{\sim}{\mu} \in W$ vs $\underset{\sim}{\mu} \in V$.

Show that
$$\lambda = \left[\frac{\| P_{V^\perp} \underset{\sim}{Y} \|^2}{\| P_{W^\perp} \underset{\sim}{Y} \|^2} \right]^{n/2}$$

(b) Show that the test
$$\varphi(F) = \begin{cases} 1 & \text{if } F > F^{\alpha}_{p-k,\, n-p} \\ 0 & \text{o.w.} \end{cases}$$
is the likelihood ratio test where
$$F = \frac{\| P_{V|W} \underset{\sim}{Y} \|^2 / p-k}{\| P_{V^\perp} \underset{\sim}{Y} \|^2 / n-p}$$

2. Suppose that $\underset{\sim}{Y} \sim N_n(\underset{\sim}{\mu}, \Sigma)$. Let $\underset{\sim}{Y} = \begin{pmatrix} \underset{\sim}{Y}_1 \\ \underset{\sim}{Y}_2 \end{pmatrix}$, $\underset{\sim}{\mu} = \begin{pmatrix} \underset{\sim}{\mu}_1 \\ \underset{\sim}{\mu}_2 \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$

where $\underset{\sim}{Y}_1$ and $\underset{\sim}{\mu}_1$ are $p \times 1$, and Σ_{11} is $p \times p$. Show that

(a) $\underset{\sim}{Y}_1 \sim N_p(\underset{\sim}{\mu}_1, \Sigma_{11})$, $\underset{\sim}{Y}_2 \sim N_{n-p}(\underset{\sim}{\mu}_2, \Sigma_{22})$

(b) $\underset{\sim}{Y}_1$ and $\underset{\sim}{Y}_2$ are indep. iff $\Sigma_{12} = 0$

(c) $\underset{\sim}{Y}_1 \mid \underset{\sim}{Y}_2 \sim N_p(\underset{\sim}{\mu}_1 + \Sigma_{12}\Sigma_{22}^{-1}(\underset{\sim}{Y}_2 - \underset{\sim}{\mu}_2),\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$ if $\Sigma_{22} > 0$