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（榮譽第一）博士班

國立政治大學八十四學年度第二學期資格考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

考試科目 Course	數理統計	開課系級 Dept, & Class	博士班	日期 Date, Period	月 日 第 節	試題編號 Course No.	

1. Let $X_1, X_2, \cdots, X_n$ be independent, identically distributed random variables each having probability density function $f(x;\theta) = \frac{1}{\theta} x^{\frac{1}{\theta}-1}$ for $0 < x < 1$, where $\theta > 0$

(8分) (a) Find the distribution of $\ln \prod_{i=1}^{n} X_i^{-\frac{1}{2\theta}}$

(8分) (b) Show that the maximum likelihood estimator of θ is $\hat{\theta} = \frac{1}{n} \ln \prod_{i=1}^{n} X_i^{-1}$

(8分) (c) Show that $\ln \prod_{i=1}^{n} X_i^{-1}$ is a complete sufficient statistic

(8分) (d) Is $\hat{\theta} = \frac{1}{n} \ln \prod_{i=1}^{n} X_i^{-1}$ a minimum variance unbiased estimator of θ? why, or why not?

(8分) (e) State carefully a result concerning the asymptotic distribution of $\hat{\theta}$.

(8分) (f) Find the likelihood ratio statistic for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ and express it as a function of $\hat{\theta}$. Show that the likelihood ratio test of significance level α reject H_0 if $\hat{\theta} \le c_1$ or $\ge c_2$, where c_1 and c_2 are constants. Give two equations from which these two constants could be found

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2. Let $X_1, X_2, \cdots, X_n$ be independent $N(\theta, 1)$ variables and consider the problem of estimating θ using squared error loss. Suppose θ has a prior $N(a, a)$ distribution, where $a > 0$ is known

(8分) (a) Find δ_a, the Bayes estimate of θ, and the corresponding Bayes risk ρ_a

(8分) (b) Determine the limit rule $\delta = \lim_{a \to \infty} \delta_a$ and show that it has constant Bayes risk ρ.

(7分) (c) Find $\rho_\infty = \lim_{a \to \infty} \rho_a$

(7分) (d) Is δ minimax? Why, or why not?

(7分) (e) Is δ admissible? Why, or why not?

(over)

③ 考

（密暑第一）

國立政治大學八十 學年度第 學期 考試試題 第 頁

考試科目	數統	開課系級		日期節次	第 月 日 節	試題編號	

3. If the probability density function of a continuous distribution is given by

$$f(x;\theta) = \frac{\theta^{\lambda}}{\Gamma(\lambda)} x^{\lambda-1} e^{-\theta x}$$

for $x > 0$, where λ is known and $\theta > 0$. A random sample of size n is taken from this distribution and, based on this sample, a test of $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ is required.

(18分) (a) Find the uniformly most powerful test of size α of H_0 against H_1 using the monotone likelihood ratio property.

(17分) (b) Show that when $\lambda = \frac{1}{n}$, the power function of the test is $1-(1-\alpha)^{\theta/\theta_0}$.