

a) Consider the regression model $\underline{Y} = \underline{X}\underline{B} + \underline{\varepsilon}$ where $\underline{Y}, \underline{\varepsilon}$ are vectors of observations and errors, respectively, where \underline{X} is an $n \times p$ matrix of predictor variables and where \underline{B} is a $p \times 1$ vector of parameters to be estimated. If the restriction $\underline{C}\underline{B} = \underline{d}$ holds, where \underline{C} is $q \times p$ ($q < p$) and of full row rank, find the least squares estimate $\hat{\underline{B}}$ in terms of $\underline{b} = (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y}$, \underline{X} , \underline{d} and \underline{C} . (20 points)

b) Now consider the linear hypothesis $\underline{C}\underline{B} = \underline{0}$. Using (a) or otherwise, find the "Sum of Squares due to H_0 ". (20 points)

c) Show that, for the data set below, $SS(H_0: B_1 = B_2)$ has value 24.5 (20 points)

$$\underline{X} = \begin{bmatrix} 1 & x_1 & x_2 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix}, \quad \underline{Y} = \begin{bmatrix} 22 \\ 20 \\ 27 \\ 29 \\ 24 \end{bmatrix}$$

2. The italicized portion of each of the following statements is either True or False. Indicate whether you feel True or False is the correct answer to each statement and provide a short justification. No credit will be answered without justification. (20 points)

a) Let T be a random variable having a t -distribution with m degree of freedom. Let G be a random variable having an F distribution with $(1, m)$ degrees of freedom.

$$\text{prob}\{T > a\} = \text{prob}(T < -a)$$

b) Consider the model $Y = b_0 + b_1X_1 + b_2X_2 + e$. Suppose you wish to test $H_0: (b_2 = 0 | b_0, b_1)$ vs $H_a: (b_2 \neq 0 | b_0, b_1)$. Assume $\text{Corr}(X_1, X_2) = 0$; $\text{Corr}(Y, X_1) = -0.8$; and $\text{Corr}(Y, X_2) = 0.1$. The null hypothesis can never be rejected. (20 points)