88學年度第二學期
（榮譽第一）

博士班

國立政治大學八十九學年度第二學期資格考試試題 第 Page

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

考試科目 Course	數理統計	開課系級 Dept, & Class	博士班	日期 Date, Period	月 日 第 節	試題編號 Course No.	1

30% 1. In a multinomial experiment with sample size 100 and 3 cells with null hypothesis $H_0: p_1 = \frac{1}{4}, p_2 = \frac{1}{2}, p_3 = \frac{1}{4}$, what is the approximate power at the alternative $p_1 = 0.2$, $p_2 = 0.6$, $p_3 = 0.2$ when the level of significance is $\alpha = 0.05$? $\alpha = 0.01$? How large a sample size is needed to achieve power 0.9 at this alternative when $\alpha = 0.05$? $\alpha = 0.01$?

30% 2. Let $X_1, \cdots, X_n$ be independent random variables having Poisson distributions with means $\exp\{\theta z_1\}, \cdots, \exp\{\theta z_n\}$, respectively, where $z_1, \cdots, z_n$ are known real numbers. Find the Cramér-Rao lower bound for the variance of an unbiased estimate of θ based on $X_1, \cdots, X_n$.

40% 3. Let $X_1, \cdots, X_n$ be a sample from the distribution with density

$$f(x|\theta_1, \theta_2) = \frac{1}{\theta_1 + \theta_2} \begin{cases} \exp\{-x/\theta_1\}, & \text{if } x > 0, \\ \exp\{+x/\theta_2\}, & \text{if } x < 0, \end{cases}$$

where $\theta_1 > 0$ and $\theta_2 > 0$ are unknown parameters.

a) Find the likelihood function in terms of the sufficient statistics, $S_1 = \sum_{j=1}^{n} X_j I(X_j > 0)$ and $S_2 = -\sum_{j=1}^{n} X_j I(X_j < 0)$. Note $S_1 \geq 0$ and $S_2 \geq 0$ but $S_1 = 0$ or $S_2 = 0$ with positive probability.

b) Find the maximum likelihood estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ as solutions of the likelihood equations.

c) Find the Fisher information matrix.

d) What is the joint asymptotic distributions of $\hat{\theta}_1$ and $\hat{\theta}_2$?

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1. Let $X_1, \ldots, X_n$ be independent, identically distributed Poisson random variables with unknown rate parameter λ. That is, the probability mass function of each X_i is

$$f(x) = \frac{\lambda^x}{x!} \exp(-\lambda), \quad x = 0, 1, \ldots$$

where $0 < \lambda < \infty$. Suppose that the X_i's are not observed, but only whether or not they are zero. That is, the observed variables are Y_i, $i = 1, \ldots, n$, where each

$$Y_i = \begin{cases} 1 & \text{if } X_i = 0 \\ 0 & \text{if } X_i > 0 \end{cases}$$

(10 分) (a) Write down the likelihood function for λ given the observed data $Y_1, \ldots, Y_n$.

(20 分) (b) Characterize the values of $Y_1, \ldots, Y_n$ for which the maximum likelihood estimate of λ exists, and find the maximum likelihood estimate of λ when it exists. Show that your estimate actually maximizes the likelihood function..

2. Let $\theta_1, \theta_2, \ldots, \theta_n$ and δ be unknown constants. Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ be mutually independent normal random variables with common unknown variance σ^2 and with means $E[X_i] = \theta_i + \delta$ and $E[Y_i] = \theta_i - \delta$. Let $D_i = X_i - Y_i$, and let $\overline{D}$ and S_D denote the sample mean and sample standard deviation of the D_i.

(20 分) (a) Show that the likelihood ratio test of $\delta = 0$ against the alternative $\delta \neq 0$ rejects the null hypothesis when $|\overline{D}/S_D|$ is large..

(10 分) (b) Find the appropriate cutoff for a specified Type I error level α.

3. Let $X_1, X_2, \ldots, X_n$ be iid from a distribution function F on R^1. Let $\hat{F}_n$ be the sample distribution given by $\hat{F}_n(t) = \frac{1}{n}\sum_{j=1}^{n} I(t - X_j)$ where $I(u) = \begin{cases} 0 & \text{if } u < 0 \\ 1 & \text{if } u \geq 0 \end{cases}$.

Fix number $t_1 < t_2 < \cdots < t_k$ and form $Z_n = \begin{pmatrix} \hat{F}_n(t_1) \\ \vdots \\ \hat{F}_n(t_k) \end{pmatrix}$.

(20 分) (a) Find the limiting distribution of $\sqrt{n}\left(Z_n - \begin{pmatrix} F(t_1) \\ \vdots \\ F(t_k) \end{pmatrix}\right)$.

(20 分) (b) Then specialize this to the case when F is the distribution of a uniform [0, 1] random variable.

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一、Let $Z_1, Z_2, \dots, Z_n$ be independently normally distributed with common variance σ^2 and means $E(Z_i)=\xi_i$ $(i=1,\dots,s)$, $E(Z_i)=0$ $(i=s+1,\dots,n)$. Show that

(i) there exist unbiased tests for testing $\xi_1 \le \xi_1^0$ and $\xi_1=\xi_1^0$ given by the rejection regions

$$\frac{Z_1-\xi_1^0}{\sqrt{\sum_{i=s+1}^{n} Z_i^2/(n-s)}} > C_0 \quad \text{and} \quad \frac{|Z_1-\xi_1^0|}{\sqrt{\sum_{i=s+1}^{n} Z_i^2/(n-s)}} > C$$

(ii) When $\xi_1=\xi_1^0$, the test statistic has the t distribution with $n-s$ degrees of freedom.

二、Let $X_1,\dots,X_m$ and $Y_1,\dots,Y_n$ be independent samples from $N(\xi,\sigma^2)$ and $N(\eta,\tau^2)$ respectively.

(i) If ξ and η are known, there exists a UMP test given by the rejection $\dfrac{\sum(Y_j-\eta)^2}{\sum(X_i-\xi)^2} \ge C$ for testing $H: \tau \le \sigma$ against $K: \sigma < \tau$

(ii) If $\tau^2=\sigma^2$, there exists a UMPI test with respect to the group $X_i'=aX_i+b$, $Y_j'=aY_j+b$ $\quad a \ne 0$ for testing $\eta=\xi$ against $\eta \ne \xi$.

命題教師 Teacher: 鄭天澤	(簽章) (Signature & date) 89 年 5 月 2 日	試題隨卷繳交