87学年度第一学期

1 Suppose the actual model of interest is the following one-way analysis of **covariance** with a **common slope** model:

$$Y_{ij} = \mu + \tau_i + \beta(x_{ij} - \bar{x}_{..}) + E_{ij}$$

with $i = 1, \ldots, a, j = 1, \ldots, n_i$, where $E_{11}, \ldots, E_{an_a}$ are *iid* normally distributed with mean 0 and unknown variance σ^2, and $\bar{x}_{..} = \sum_{i=1}^{a}\sum_{j=1}^{n_i} x_{ij}$

Suppose the researcher assumed, by mistake, the one-way model:

$$Y_{ij} = \mu^* + \tau_i^* + E_{ij}^*$$

where $E_{11}^*, \ldots, E_{an_a}^*$ are again *iid* normally distributed with mean 0 and unknown variance ${\sigma^*}^2$.

(a) Find the actual expected value of the least squares estimators of μ^* and τ_i^* (20%).

(b) For the actual (analysis of **covariance**) model, describe a multiple comparison procedure on the pairwise differences among the parameters

$$\mu + \tau_1, \ldots, \mu + \tau_a$$

What are the associated simultaneous confidence intervals? (15%)

(c) Can the researcher detect the mistake made on the model specification? How? (15%)

2 Consider the two-way with **no-interaction** model:

$$Y_{ijk} = \mu + \tau_i + \beta_j + E_{ijk}$$

with $i = 1, \ldots, a, j = 1, \ldots, b, k = 1, \ldots, n_{ij}$, where we assume $n_{ij} \geq 1$, and $E_{111}, \ldots, E_{abn_{ab}}$ are *iid* normally distributed with mean 0 and unknown variance σ^2.

For simplicity, let

$$\begin{aligned} n_{i.} &= \sum_{j=1}^{b} n_{ij} \\ n_{.j} &= \sum_{i=1}^{a} n_{ij} \\ n_{..} &= \sum_{i=1}^{a}\sum_{j=1}^{b} n_{ij} \\ \bar{Y}_{i..} &= \sum_{j=1}^{b}\sum_{k=1}^{n_{ij}} Y_{ijk} \end{aligned}$$

(a) Derive a level α test for the **null** hypothesis

$$H_0 : \tau_1 = \cdots = \tau_a = 0$$

(30%)

(b) Prove that the least squares estimators of $\tau_i - \tau_a$, $i = 1, \ldots, a-1$ are $\bar{Y}_{i..} - \bar{Y}_{a..}$, $i = 1, \ldots, a-1$, the same as in the one-way model, if

$$n_{ij} = n_{i.}n_{.j}/n_{..}$$

for all $i = 1, \ldots, a, j = 1, \ldots, b$, i.e., with proportional cells frequencies. (**Hint:** Write the model in the form of general linear model using vector and matrix notations, assume a set of side-conditions such as:

$$\sum_{i=1}^{a} n_{i.}\tau_i = \sum_{j=1}^{b} n_{.j}\beta_j = 0$$

and figure out when will the design be orthogonal.) (15%)

(c) How will your test in **(a)** be simplified, if the two-way model has proportional cells frequencies. (5%)