國立政治大學 統 計 研究所八十二學年度第二學期資格考試試題

※請將此份試題作答於第一卷。

科 目：線性模式專題

第 一 題：(30分)

1. A 2x2 factorial experiment was conducted to determine the effect of oil viscosity and temperature on the cylinder wear of a diesel engine. The four factorial combinations of the factorial experiment were assigned in random order for running on each of two engines (which would be regarded as blocks). The resulting randomized block design produced the data shown in the table. The linear model appropriate for the experiment is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_3 + \epsilon.$$

Engine (block)	Oil viscosity Actual	Oil viscosity Coded x_1	Temperatures Actual	Temperatures Coded x_2	Wear
1	5	−1	10	−1	3
	5	−1	20	1	5
	10	1	10	−1	7
	10	1	20	1	4
2	5	−1	10	−1	5
	5	−1	20	1	8
	10	1	10	−1	9
	10	1	20	1	5

(a) Estimating the parameters β_i's in the model.
(b) What does each of the $\hat{\beta}_i$'s represent?

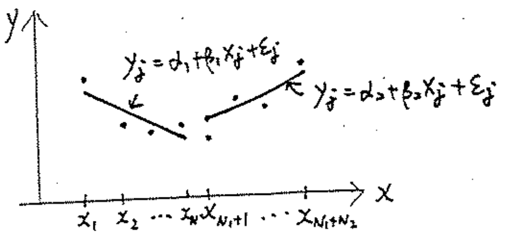
第 二 題：(20分)

2. Suppose that a regression problem is well fitted by two straight line models

$$y_j = \alpha_1 + \beta_1 x_j + \varepsilon_j \quad \text{and}$$
$$(\ j = 1, 2, \ldots, N_1\ )$$

$$y_j = \alpha_2 + \beta_2 x_j + \varepsilon_j$$
$$(\ j = N_1+1, N_1+2, \ldots, N_1+N_2\ )$$

with different intercepts and slopes, as indicated in the following figure:



Question:
Write a general linear model, for an example like

$$y_j = \delta_0 + \delta_1 x_{1j} + \ldots + \delta_p x_{pj} + \varepsilon_j,$$

to stand for the two simple linear regressions.

※請將此份試題作答於第二卷。

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科 目：線性模式專題

第三題：

Consider a linear model in which we observe $Y_{ij} = \theta + \alpha_i + \beta_j + \delta x_{ij} + \varepsilon_{ij}$, $i=1,\cdots,r$, $j=1,\cdots,c$, where $\varepsilon_{ij} \sim$ i.i.d. $N(0, \sigma^2)$, ~~and~~ x_{ij} are known constants such that $\bar{x}_{i.} = 0$ and $\bar{x}_{.j} = 0$, and $\theta, \alpha_i, \beta_j, \delta$, and σ^2 are unknown parameters such that $\sum \alpha_i = 0$ and $\sum \beta_j = 0$.

(a) Show that this model is an orthogonal design.

(b) Find the least squares estimators of $\theta, \alpha_i, \beta_j$ and δ.

(c) Find the F-test for testing that $\alpha_i = 0$ and the simultaneous confidence interval for contrasts in the α_i.

(d) Find the F-test for testing that $\delta = 0$ and a confidence interval for δ.

第四題：

Let X be a basis matrix for a subspace V in R^n.

(a) Show that $\underset{\sim}{v} \in V$ iff $\underset{\sim}{v} = X\underset{\sim}{b}$ for some vector $\underset{\sim}{b}$.

(b) Show that $X(X'X)^{-1}X'\underset{\sim}{y}$ is a projection of $\underset{\sim}{y}$ on V.

(c) Show that the projection is unique.

(d) Show that A is a projection matrix iff it is idempotent.

(Note: $\underset{\sim}{z}$ is a projection of $\underset{\sim}{y}$ on V if $\underset{\sim}{z} \in V$ and $\underset{\sim}{y} - \underset{\sim}{z} \in V^{\perp}$.)

科 目：線性模式專題

第 五 題：

Consider the model $Y = X\beta + Z\gamma + \epsilon$, where Y is n by 1, X is n by p and of full rank, Z is n by q, β is p by 1, γ is q by 1, and ϵ is $N(0, \sigma^2 I_n)$. Want to test whether $\gamma = 0$.

(1.) If Z is not a function of $X\beta$, derive the test statistics.

(2.) If $z_{ij} = z_{ij}(X\beta)$ for all i and j, and the functional forms are assumed known, derive the test statistic.

第 六 題：

Consider the model $Y = X\beta + \epsilon$, where Y is n by 1, X is n by p, β is p by 1, and ϵ is n by 1 and is random with $E(\epsilon) = 0$, $Var(\epsilon) = \sigma^2 I_n$.

(1.) Show that if an n by n matrix A satisfies $A = A^T = A^2$ and $AX = X$, then for an n by 1 vector c, $E[c^T AY] = E[c^T Y]$ and $Var[c^T AY] \leq Var[c^T Y]$.

For (2) and (3) assume $n = p = 3$.

(2.) Suppose $$X = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

Find $A \neq I_3$ such that $A = A^T = A^2$ and $AX = X$.

(3.) Use (1) and (2) to find a 3 by 1 b such that $E[b^T Y] = \beta_1 + \beta_3$, and $Var[b^T Y] < \sigma^2$. Verify your answer.