國立政治大學 統計研究所 八十學年度第二學期資格考試試題

科目：數理統計

第一題：

Let X be a Binomial random variable with $n = \#$ of trials and $p =$ probability of a success in each trial. The problem is to estimate p with a loss function $L(\delta, p) = (\delta - p)^2 / pq$, where $q = 1-p$ and $\delta(X)$ is an estimator of p.

(a) Find a minimax estimator for p. (10%)

(b). Taking Beta distribution with parameters α and β, as a prior distribution for p, find for what values of α and β, the above minimax estimator will be a Bayes procedure. (15%)

第二題：

Let $X_1, \cdots X_n$ be i.i.d. random variables with common distribution $N(\mu, \sigma^2)$, where $-\infty < \mu < \infty$ and $0 < \sigma^2 < \infty$. The group G considered is that of the real affine transformations; i.e. $G = \{ g_{\alpha,\beta} \mid g_{\alpha,\beta}(x) = \alpha + \beta x,\ -\infty < \alpha < \infty,\ 0 < \beta < \infty \}$. Find the minimal mean-squared error equivariant estimators of μ and σ^2, respectively. (25%)

第三題：3. Let $X_1, X_2, \cdots, X_n$ be independent, $X_j = 0$ or 1 with respective probabilities $1-p$, p. Let $\phi(x) = \phi(x_1, \cdots, x_n)$ be the critical function of a test for testing whether p is "small" or "large". Let $0 < p_1 < p_2 < 1$,

$$\alpha_1(\phi) = \sup_{p \le p_1} E_p \phi, \quad \alpha_2(\phi) = \sup_{p \ge p_2} E_p(1-\phi)$$

(a). Find a test which minimizes $\alpha_1(\phi)$ and $\alpha_2(\phi)$ subject to the condition $\alpha_1(\phi) = c\,\alpha_2(\phi)$, where c is a fixed positive number. (10%)

(b) Is this test unique? Why? (5%)

4. (I). The following problems are concerned with tests based on a sample $x_1, \cdots, x_n$ which are the observed values of a random vector $(X_1, \cdots, X_n)$ whose distribution depends on a parameter θ contained in Ω, a subset of a Euclidean space. The set Ω_0 is a subset of Ω. (a). State definitions of the followings: (i) Unbiased test of size α for testing the hypothesis $\theta \in \Omega_0$ against the alternative $\theta \in \Omega - \Omega_0$. (ii) Similar test of size α for testing hypothesis $\theta \in \Omega_0$. (10%)

第四題：

(b) State a sufficient condition under which an unbiased test for testing the hypothesis $\theta \in \Omega_0$ against $\theta \in \Omega - \Omega_0$ is a similar test for testing the hypothesis $\theta \in \Omega^*$, where Ω^* is a certain subset of Ω_0. Prove your statement. (10%)

(II). Let $X_1, X_2, \cdots, X_m; Y_1, \cdots, Y_n$ be $m+n$ independent random variables such that X_i has the discrete frequency function $f(x, \theta_1)$ and each Y_j has the discrete frequency function $f(x, \theta_2)$, where

$$f(x, \theta) = (1-\theta)\theta^x, \quad x = 0, 1, 2, \cdots$$

and the parameters θ_1 and θ_2 are unknown except that $0 < \theta_i < 1$, $i = 1, 2$. Find the uniformly most powerful unbiased test for testing the hypothesis $\theta_1 \le \theta_2$ against the alternative $\theta_1 > \theta_2$. (15%)