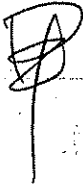


第一部份：

(-) Let X_1, \dots, X_n be a random sample (i.i.d.)
20% on X , where $X \sim N(\mu, 1)$.

Ⓐ Find the expected value θ of $Y = \exp(X)$,
i.e. $\theta = E(Y) = ?$

Ⓑ Find the Cramer-Rao lower bound
for the variance of an unbiased estimate of θ
(given X_1, \dots, X_n).

Ⓒ Find the uniformly minimum variance
unbiased estimate T of θ , i.e. $T = \hat{\theta} = ?$

Ⓓ Show that $\text{Var}(T) = \theta^2(e^{\frac{1}{n}} - 1)$ and
verify that this variance is bigger
than the lower bound found in Ⓑ

(=) Let X_1, \dots, X_n be independent $N(\mu, 1)$ r.v.'s.

15% Consider the problem of estimating μ using squared error loss where μ has a prior $N(0, \theta)$ distribution with $\theta > 0$ being known.

(a) Find δ_θ , the Bayes estimate of μ , and the corresponding Bayes risk ρ_θ .

(b) Find the estimate δ being defined as $\delta = \lim_{\theta \rightarrow \infty} \delta_\theta$ and show that δ has constant risk.

(c) Is δ a minimax estimate of μ ?

Is δ admissible? Why or why not?

(≡) Let X_1, \dots, X_n be a random sample from
10% an exponential distribution with the p.d.f.

$$f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0, \quad (\theta > 0)$$

Let m denote the median of this distribution.

(a) Find the MLE, \hat{m} of m .

(b) Show that \hat{m} is a best linear unbiased estimator.