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國立政治大學統計研究所八十學年度第一學期學科考試試題

科目：數理統計專題

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第一部份：甲

(一) Let $X_1, \ldots, X_n$ be a random sample (i.i.d.)

20% on X, where $X \sim N(\mu, 1)$.

ⓐ Find the expected value θ of $Y = \exp(X)$,

i.e. $\theta = E(Y) = ?$

ⓑ Find the Cramer-Rao lower bound for the variance of an unbiased estimate of θ (given $X_1, \ldots, X_n$).

ⓒ Find the uniformly minimum variance unbiased estimate T of θ, i.e. $T = \hat{\theta} = ?$

ⓓ Show that $Var(T) = \theta^2(e^{\frac{1}{n}} - 1)$ and verify that this variance is bigger than the lower bound found in ⓑ

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(二) Let $X_1, \dots, X_n$ be independent $N(\mu, 1)$ r.v.'s.

15% Consider the problem of estimating μ using squared error loss where μ has a prior $N(0, \theta)$ distribution with $\theta > 0$ being known.

ⓐ Find δ_θ, the Bayes estimate of $\cancel{\theta}\ \mu$, and the corresponding Bayes risk ρ_θ.

ⓑ Find the estimate δ being defined as $\delta = \lim_{\theta \to \infty} \delta_\theta$ and show that δ has constant risk.

ⓒ Is δ a minimax estimate of $\cancel{\theta}\ \mu$? Is δ admissible? Why or why not?

(三) Let $X_1, \dots, X_n$ be a random sample from

10% an exponential distribution with the p.d.f.

$$f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0, \ (\theta > 0)$$

Let m denote the median of this distribution.

ⓐ Find the MLE, $\hat{m}$ of m.

ⓑ Show that $\hat{m}$ is a best (uniformly