(榮譽第一)
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NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

考試科目 Course	數統資格考	開課系級 Dept, & Class	統計系博士班	日期 Date, Period	月 日 第 節	試題編號 Course No.	

33% 1. Let Θ have a distribution Λ, and let P_θ denote the conditional distribution of X given θ. Consider the estimation of $g(\theta)$ when the loss function is squared error. Then no unbiased estimator $\delta(X)$ can be a Bayes solution unless

$$E[\delta(X)-g(\Theta)]^2=0$$

where the expectation is taken with respect to a variation in both X and Θ.

33% 2. Let θ be a real parameter, and let the random variable X have probability density $P_\theta(x)$ with monotone likelihood ratio in $T(x)$. Define

$$(1)\qquad \phi(x)=\begin{cases}1 & \text{when } T(x)>C\\ \gamma & \text{when } T(x)=C\\ 0 & \text{when } T(x)<C\end{cases}$$

where C and γ are determined by

$$(2)\qquad E_{\theta_0}\phi(x)=\alpha.$$

Show that, for testing $H:\theta\le\theta_0$ against $K:\theta>\theta_0$, the family of tests given by (1) and (2) with $0\le\alpha\le1$ is essentially complete provided the loss function $L(\theta,d)$ satisfies

$$L_1(\theta)-L_0(\theta)\gtrless 0 \quad \text{as} \quad \theta\lesseqgtr\theta_0, \text{ where}$$

$$L(\theta,d)=\begin{cases}L_0(\theta) & \text{when the decision } d=d_0 \text{ (i.e. } H \text{ is accepted)}\\ L_1(\theta) & \text{when the decision } d=d_1 \text{ (i.e. } H \text{ is rejected)}\end{cases}$$

34% 3. Let $X_1, X_2, \cdots, X_n$ be independent, with $X_i\sim\Gamma(g,b)$, where g and b are both unknown. (Note that if $X\sim\Gamma(g,b)$, then the density is $\frac{1}{\Gamma(g)\,b^g}x^{g-1}e^{-x/b}$, $0<x\le\infty$).

(a) Find the UMVU (uniformly minimum variance unbiased) estimator of gb.

(b) Find the UMVU of $g^n b^n$.